



## Analysis of the Convergence Behaviour of Double-Step and Triple-Step Iteration Schemes

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### Abstract

Fixed point iterative methods are commonly used to solve equations arising from physical models. However, a comprehensive numerical analysis of these methods is often lacking. In this work, we provide a rigorous analysis of the convergence behavior and associated rates of convergence of selected iterative approaches under variational conditions in Banach space with the aid of Z-operator. Furthermore, the study examines the existence of fixed points generated by the S-iteration and Noor iterative schemes.

**Keywords:** Fixed point; Banach space; Convergence; Normed space; Metric Space; Banach fixed point

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### Introduction

From a mathematical perspective, a fixed point represents a point which does not change upon application of a map. Also referred to as an invariant point or fix point, a value  $k$  serves as a fixed point of a value of the function  $f$  at  $x$  precisely when  $f(k) = k$ . Many mathematical and applied problems may be modeled as fixed point problems, and algorithms toward locating such points play a significant role in numerous applications (Berinde, 2007).

From algorithmic mathematics, iterative methods refer to analytical procedures which generate sequences of increasingly accurate approximate solutions for various problems. A particular application of such a method, including conditions for termination, constitutes an iterative procedure. An iterative technique is said to converge if the generated sequence approaches a limit from a given value approximation.

When solving equations or systems of equations, iterative methods begin with a trial value and produce iterative estimates of the

solution. Unlike direct approaches, which aim to compute exact solutions in a finite number of steps (e.g., Gaussian elimination for linear systems), iterative methods are especially valuable for nonlinear problems and large-scale linear problems where direct methods are computationally expensive or impractical. The concept of fixed points in Banach spaces is particularly important in the analysis of iterative transformation systems, especially with the application contraction mappings. Banach's fixed point theorem performs a central part in proving the occurrence of solutions via Picard iteration and also employed in a proof of the inverse function theorem (Banach, 1922).

However, when contraction conditions are weakened, Picard iteration may no longer converge, necessitating alternative iterative procedures. In such contexts—where the operator is merely non-expansive—the Banach space framework is utilized. Common iterative schemes under these conditions include Mann and Ishikawa iterations, with

convergence results detailed in Chidume (1994) and Koti *et al.* (2013).

An alternative method, known as the S-iteration process, was proposed by Agarwal *et al.* (2007) in normed linear spaces and is distinct from the Mann and Ishikawa schemes. This paper investigates the strong convergence analysis of the S-iteration and Noor iteration methods in convex Banach spaces. The primary objective is to create and compare the strong convergence and convergence rates of iterative techniques with errors, utilizing Zamfirescu operators. Furthermore, we develop a unified iterative scheme that encompasses several existing methods.

#### **Preliminary Results**

The following Lemma and Theorem are required to prove our main results.

**Lemma 1:** (Berinde, 2002): Let  $\delta$  be a real number such that  $\delta \in [0, 1]$  and  $\{e_n\}_{n=0}^{\infty}$  is a sequence of non-negative numbers such that  $\lim_{n \rightarrow \infty} e_n = 0$ ; then, for any sequence of positive numbers  $\{U_n\}_{n=0}^{\infty} \subset X$  satisfying

$$U_{n+1} \leq \delta U_n + e_n \text{ for all } n \in \mathbb{N} \quad (1)$$

Moreover, the Z-operator used the following theorem to prove the strong convergence of Picard iteration.

#### **Theorem 1:**

(Zamfirescu, 1972): Let  $(X, d)$  be a complete metric space and let  $T : X \rightarrow X$  be a mapping. Suppose that there exist constants  $a, b, c \in [0, 1]$  such that for every  $x, y \in X$ , at least one of the following conditions holds:

1.  $d(Tx, Ty) \leq ad(x, y)$ ;
2.  $d(Tx, Ty) \leq b[d(x, Tx) + d(y, Ty)]$ ;  
and
3.  $d(Tx, Ty) \leq c[d(x, Ty) + d(y, Tx)]$

Then,  $T$  has a unique fixed point  $x^* \in X$  (i.e.,  $Tx^* = x^*$ ) and for any initial point  $x_0 \in X$ , the iterative sequence defined by  $x_{n+1} = Tx_n$ .

The three-step iteration which is called Noor iteration was introduced by Noor (2000) as follows:

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Ty_n$$

$$y_n = (1 - \beta_n)x_n + \beta_n Tz_n$$

$$z_n = (1 - \gamma_n)x_n + \gamma_n Tx_n, \text{ for all } n \geq \mathbb{N} \quad (2)$$

where  $\{\alpha_n\}_{n=0}^{\infty}$ ,  $\{\beta_n\}_{n=0}^{\infty}$  and  $\{\gamma_n\}_{n=0}^{\infty}$  are sequences of positive numbers in  $[0, 1]$  such that their sum is infinity.

Another iteration called S-iteration process, was introduced by Agarwal *et al.* (2007). It is defined in a normed linear space as follows: If  $x_0 \in E$ ,  $E$  is a normed space, the sequence  $\{x_n\}$  in  $E$  is defined as:

$$x_{n+1} = (1 - \alpha_n)y_n + \alpha_n Ty_n$$

$$y_n = (1 - \beta_n)x_n + \beta_n Tx_n, \text{ for all } n \geq 0 \quad (3)$$

where  $\alpha_n, \beta_n \in [0, 1]$  with  $\sum \alpha_n = \infty$ .

Mann (1953) defined a more general iteration in a Banach space setting satisfying quasi-non expansive operators. The Mann iteration is given as:

$$x_n = (1 - \alpha_n)x_{n-1} + \alpha_n Tx_{n-1} \quad (4)$$

where  $\alpha_n$  is a sequence of positive numbers  $[0, 1]$ . By putting  $\alpha_n = 1$  into (4) yields Picard iteration.

Liu (1995) introduced the concept of Mann iteration process with errors by the sequence  $\{x_n\}$  defined as follows:

$$x_n = (1 - \alpha_n)x_{n-1} + \alpha_n T x_{n-1} + U_n \quad (5)$$

Where  $\{x_n\}$  satisfies  $\sum \|U_n\| < \infty$ .

### Main Results

Within this section, we examine the convergence and rate of convergence of two iterative schemes. Our main results are formulated using Z-operators in Banach spaces, specifically considering iterative schemes with errors terms.

Additionally, we provide computational results comparing the rates of convergence of S-iteration and Noor iteration methods. The key findings are established by the following theorems.

### Theorem 2.

Let  $M$  be a non-empty subset of a normed linear space  $(E, \|\cdot\|)$  and let  $K: M \rightarrow M$  be a self-map satisfying the Zamfirescu condition for all  $u, v \in M$ . Let  $\{u_n\}_{n=0}^\infty$  be the sequence generated by the S-iterative scheme. Suppose that:

Given arbitrary  $u, v \in M$ ,

$$\|K(u) - K(v)\| \leq a\|u - v\| + 2a(\|u - Ku\|)$$

where  $a \in [0, 1)$  since  $p_1$  along with  $p_2$  are fixed point,

$$Kp_1 = p_1 \text{ and } Kp_2 = p_2.$$

Therefore,

$$\|p_1 - p_2\| = \|Kp_1 - p_2\| \leq a\|p_1 - p_2\| + 2a(\|p_1 - K(p_1)\|)$$

But  $\|p_1 - Kp_1\| = 0$  because  $p_1$  is a fixed point. Thus,

$$\|p_1 - p_2\| \leq a\|p_1 - p_2\|$$

Subtracting  $a\|p_1 - p_2\|$  from both sides;

$$(1 - a)\|p_1 - p_2\| \leq 0$$

Since  $(1 - a) > 0$ , it follows that  $\|p_1 - p_2\| \leq 0$ . However, norms are nonnegative, so the only possibility is:

$$\|p_1 - p_2\| = 0 \Rightarrow p_1 = p_2$$

This contradicts our initial assumption that  $p_1 \neq p_2$

The resulting contradiction shows that our assumption is false.

Therefore, the mapping  $K$  has a unique fixed point  $p$ .

(2) To show the S-iterative method generates a sequence  $\{u_n\}$  converges strongly to the unique fixed point  $p$  from the mapping  $K$ , that is,

$$\lim_{n \rightarrow \infty} u_n = p.$$

Given:

The mapping  $K: M \rightarrow M$  satisfies the condition:

$$\|K(u) - K(v)\| \leq \|u - v\| + 2\delta \|u - Ku\| \quad \forall u, v \in C, \quad (6)$$

such that  $\delta \in [0, 1)$ .

From a previous result of Lemma 1, it follows that:

$$\|K(u_n) - q\| \leq \delta \|u_n - q\|, \quad (7)$$

where  $q$  is left fixed by  $K$ .

Establishing the Recursive relation:

•  $F(K) \neq \emptyset$  (i.e. there exists at least a point fixed by  $K$ ),

•  $\sum_{n=1}^\infty a_n = \infty$ ,

•  $\|V_n\| = 0$ .

Then:

(1)  $K$  defined by Zamfirescu operator has unique fixed point  $p$ ;

(2) the S-iterative scheme converges strongly to a fixed point of  $K$ .

Proof:

(1) To prove that  $K$  possesses a single fixed point under the given condition. Suppose the function possesses two different fixed points  $p_1, p_2 \in F(K)$  such that  $p_1 \neq p_2$ . This implies

$$\|p_1 - p_2\| > 0$$

Let  $a_n = \|u_n - p\|$ , where  $K$  has a unique fixed point, namely  $p$ . Using triangle inequality and given conditions, we aim to find a recursive relationship for  $a_n$ .

Applying the mapping condition:

From condition (6), for  $u = u_n$  and  $v = p$ , it follows that:

$$\|K(u) - K(p)\| \leq \|u_n - p\| + 2\delta \|u_n - K(u_n)\|.$$

Since  $K(p) = p$ , this simplifies to:

$$\|K(u) - p\| \leq \|u_n - p\| + 2\delta \|u_n - K(u_n)\|.$$

Relating  $u_{n+1}$  to  $u_n$ :

In the S-iterative scheme,  $u_{n+1}$  is defined in terms of  $u_n$  and  $K(u_n)$ . While the exact form of the S-iteration scheme is not specified here, it generally involves combination of  $u_n$  and  $K(u_n)$ , such as:

$$u_{n+1} = (1 - a_n)u_n + a_n K(u_n),$$

such as  $a_n \in [0, 1]$ . This remains similar to the Mann iteration scheme.

Applying this, it can be written as:

$$\|u_{n+1} - p\| = \|(1 - a_n)(u_n - p) + a_n K(u_n) - p\|,$$

Applying the inequality of the triangle:

$$\|(u_{n+1} - p)\| \leq (1 - a_n) \|u_n - p\| + a_n \|K(u_n) - p\|.$$

Substituting the earlier bound for  $\|K(u_n) - p\|$

$$\|u_{n+1} - p\| \leq (1 - a_n) \|u_n - p\| + a_n (\|u_n - p\| + 2\delta \|u_n - K(u_n)\|).$$

Simplifying:

$$\|u_{n+1} - p\| \leq \|u_n - p\| + 2\delta a_n \|u_n - K(u_n)\|.$$

Bounding  $\|u_n - K(u_n)\|$ :

From (Lemma 1), we have:

$$\|K(u_n) - p\| \leq \delta \|u_n - p\|.$$

Using the inequality of the triangle:

$$\|u_n - K(u_n)\| \leq \|u_n - p\| + \|K(u_n) - p\| \leq \|u_n - p\| + \delta \|u_n - p\| = (1 + \delta) \|u_n - p\|.$$

Combining the inequalities: Substituting the bound for  $\|u_n - K(u_n)\|$  into the inequality for  $\|u_{n+1} - p\|$ :

$$\|u_{n+1} - p\| \leq \|u_n - p\| + 2\delta a_n (1 + \delta) \|u_n - p\| = [1 + 2\delta a_n (1 + \delta)] \|u_n - p\|.$$

Establishing convergence:

since  $\delta \in [0, 1]$  and  $a \in [0, 1]$ , the term  $2\delta a_n (1 + \delta)$  is less than 2. Therefore, the coefficient  $[1 + 2\delta a_n (1 + \delta)]$  is less than a constant greater than 1, depending on  $\delta$  and  $a_n$ .

However, to ensure convergence, we need this coefficient to be less than 1. This suggests that additional conditions on  $a_n$  and  $\delta$  are necessary, such as choosing  $a_n$  sufficiently small or  $\delta$  close to 0.

Assuming such conditions are met, we can write:

$$\|(u_{n+1} - p)\| \leq r \|u_n - p\|,$$

in which  $r \in (0, 1)$ , By inductive argument:  $\|(u_n - p)\| \leq r_n \|u_n - p\| \rightarrow 0$  as  $n \rightarrow \infty$ .

Under the given conditions and appropriate choices of  $a_n$  and  $\delta$ , the sequence  $u_n$  produced via the S-iterative scheme converges in the strong sense towards single solution  $p$  of  $K$ ; which means,  $\lim_{n \rightarrow \infty} u_n = p$ .

Thus, the desired result is proved.

### **Theorem 3.**

Let  $M$  be a non-empty subset of a normed linear space  $(E, \|\cdot\|)$  and let  $K: M \rightarrow M$  be a self-mapping satisfying the Zamfirescu condition for all  $u, v \in M$ . Consider  $\{u_n\}_{n=0}^{\infty}$  produced via Noor iterative scheme defined by equation (2). Suppose that:

- The collection of points fixed by  $K$ , denoted  $F(K)$ , is not empty, which means,  $F(K) \neq \emptyset$ .
- The series  $\sum_{n=1}^{\infty} a_n = \infty$ ,  $\|V_n\|$  converges, that is  $\sum_{n=1}^{\infty} a_n = \infty$ ,  $\|V_n\| < \infty$ .

Then:

(1) Existence and uniqueness: The mapping  $K$  defined in Zamfirescu operator admits a unique fixed point  $p \in M$ .

(2) Strong convergence: Sequence  $u_n$ , obtained via the Noor iterative process, approaches to a fixed point of  $p$ , in which,  $\lim_{n \rightarrow \infty} u_n = p$ .

*Proof:*

(1) Our aim is to demonstrate that the operator  $K$  fulfills Zamfirescu operator possesses a uniquely determined fixed point.

Assume, for contraction, that there exist two distinct points  $p_1, p_2 \in F_K$  such as

$$p_1 \neq p_2 \text{ and } \|(p_1 - p_2)\| > 0.$$

However:

$$0 < \|(p_1 - p_2)\| = Kp_1 - Kp_2 \leq 2a\|(p_1 - Kp_1)\| + a\|(p_1 - p_2)\|$$

It leads to:

$$\|(p_1 - p_2)\| \leq a\|(p_1 - p_2)\|$$

Subtracting  $a\|(p_1 - p_2)\|$  from both sides:

$$(1 - a) \|(p_1 - p_2)\| \leq 0$$

Since  $1 - a > 0$  and norms are non-negative, this implies:

$$\|p_1 - p_2\| = 0$$

Therefore,  $p_1 = p_2 = p$ , contradicting our assumption that  $p_1 \neq p_2$ .

Hence,  $K$  has a unique fixed point  $p$ .

(2) We aim to demonstrate that there is strong convergence of  $u_n$  to the fixed point  $p$  of the mapping  $K$ , i.e;  $\lim_{n \rightarrow \infty} u_n = p$

From Zamfirescu operator we have:

$$\|Ku - Kv\| \leq \delta\|u - v\| + 2\delta\|u - Ku\| \quad \forall u, v \in \mathbb{C}$$

Additionally, from Lemma 1, we get

$$\|(Ku_n - q)\| \leq \delta\|u_n - q\|$$

Applying this to equation (2), we get

$$\begin{aligned} \|u_{n+1} - p\| &\leq \|(1 - a_n)u_n + a_nKv_n + U_n - p\| \\ &= \|(1 - a_n)(u_n - p) + a_n(Kv_n - p) + U_n\| \\ &\leq (1 - a_n)\|u_n - p\| + a_n\|Kv_n - p\| + \|U_n\| \\ &\leq (1 - a_n)\|u_n - p\| + a_n\delta\|v_n - p\| + \|U_n\| \end{aligned} \quad (8)$$

Similarly, from Lemma 1, we have:

$$\begin{aligned} \|v_n - p\| &\leq \|(1 - b_n)u_n + b_nKw_n + V_n - p\| \\ &= \|(1 - b_n)(u_n - p) + b_n(Kw_n - p) + V_n\| \\ &\leq (1 - b_n)\|u_n - p\| + b_n\|Kw_n - p\| + \|V_n\| \\ &\leq (1 - b_n)\|u_n - p\| + b_n\delta\|w_n - p\| + \|V_n\| \end{aligned} \quad (9)$$

From Lemma 1, we also obtain:

$$\begin{aligned} \|w_n - p\| &\leq \|(1 - c_n)u_n - p + c_nKu_n + f_n\| \\ &= \|(1 - c_n)(u_n - p) + c_n(Ku_n - p) + f_n\| \\ &\leq [(1 - c_n)\|u_n - p\| + c_n\delta\|Ku_n - p\|] + \|f_n\| \\ &\leq [(1 - c_n) + c_n\delta]\|u_n - p\| + \|f_n\| \end{aligned} \quad (10)$$

Substituting equation (10) into (9), we derive:

$$\begin{aligned} \|v_n - p\| &\leq (1 - b_n)\|u_n - p\| + b_n\delta(1 - c_n + c_n\delta)\|u_n - p\| + \|f_n\| + \|V_n\| \\ &= [(1 - b_n) + b_n\delta(1 - c_n + c_n\delta)]\|u_n - p\| + \|f_n\| + \|V_n\| \end{aligned} \quad (11)$$

Similarly, substituting equation (11) into equation (8), it follows that:

$$\begin{aligned} \|u_{n+1} - p\| &\leq (1 - a_n)\|u_n - p\| + a_n\delta(1 - b_n + b_n\delta(1 - c_n + c_n\delta))\|u_n - p\| + \|f_n\| \\ &\quad + \|V_n\| + \|U_n\| \\ &= [(1 - a_n) + a_n\delta(1 - b_n + b_n\delta - b_nc_n\delta + b_nc_n\delta^2)]\|u_n - p\| + \|f_n\| + \|V_n\| \\ &\quad + \|U_n\| \end{aligned}$$

(12)

Letting  $\|e_n\| = \|f_n\| + \|V_n\| + \|U_n\|$ , and applying Lemma 1 under the theorem's conditions, we conclude that:

$$\lim_{n \rightarrow \infty} \|u_{n+1} - p\| = 0.$$

Hence,  $u_{n+1} \rightarrow p \in F_K$ , indicating  $u_n$  approaches  $p$  with respect to the norm topology. This concludes the proof.

**Corollary 1.**

Let  $M$  be a non-empty subset of a normed linear space,  $(E, \|\cdot\|)$ , each  $u, v \in M$ . Let  $K : M \rightarrow M$  be a self-map of  $F$  satisfying the Zamfirescu operator for each  $u, v \rightarrow F$  where  $F_K \neq \emptyset$ ,  $\sum_{n=1}^{\infty} \alpha_n = \infty$ ,  $\|V_n\| = 0$ . Then

- (i)  $K$  defined by Lemma 1 has a unique fixed point;
- (ii) the  $S$ -iterative scheme defined in (3) converges strongly to  $p$  of  $K$ ;
- (iii) the Noor iterative scheme defined in (2) converges strongly to  $p$  of  $K$ .

*Proof:*

The corollary is an immediate consequence of Theorems 2 combined with 3 where  $\beta_n = 1$ .

**Numerical Example**

The function  $f(u) = (1 - u)^2$  possesses multiple zeros. Using the initial guess  $u = 0.9$  and setting  $\alpha_n = \beta_n = \frac{1}{2}$ , the merging behavior of the iterative methods toward the precise solution point  $p = 0.38196601$  is compared and illustrated within the Table 1.

Furthermore, the error estimates  $e_n = \|q - u_n\|$ , recorded at interval of five iterations based on the data from Table 1, are provided in Table 2.

**Table 1: The outcomes across multiple iterations of the sample**

| N  | S-Iteration | Noor iteration |
|----|-------------|----------------|
| 0  | 0.90000000  | 0.90000000     |
| 1  | 0.15351250  | 0.53059611     |
| 2  | 0.51786791  | 0.41092632     |
| 3  | 0.31143842  | 0.38648213     |
| 4  | 0.42141796  | 0.38263205     |
| 5  | 0.36076612  | 0.38396925     |
| 6  | 0.39361130  | 0.38225969     |
| 7  | 0.37564509  | 0.38200889     |
| 8  | 0.38541937  | 0.38197227     |
| 9  | 0.38008599  | 0.38196692     |
| 10 | 0.38299148  | 0.38196614     |
| 11 | 0.38140725  | 0.38196603     |
| 12 | 0.38227064  | 0.38196601     |
| 13 | 0.38179998  |                |
| 14 | 0.38205652  |                |
| 15 | 0.38191668  |                |
| 16 | 0.38199290  |                |
| 17 | 0.38195135  |                |
| 18 | 0.38197400  |                |
| 19 | 0.38196166  |                |
| 20 | 0.38196839  |                |
| 21 | 0.38196472  |                |
| 22 | 0.38196672  |                |

|    |            |
|----|------------|
| 23 | 0.38196563 |
| 24 | 0.38196622 |
| 25 | 0.38196590 |
| 26 | 0.38196607 |
| 27 | 0.38196598 |
| 28 | 0.38196603 |
| 29 | 0.38196600 |
| 30 | 0.38196601 |

**Table 2: The result iteration-wise error estimation of the example.**

| N | S-Iteration | Noor iteration |
|---|-------------|----------------|
| 0 | 0.00211999  | 0.00200323     |
| 1 | 0.00102547  | 0.00000013     |
| 2 | 0.00002689  | 0.00000000     |
| 3 | 0.00000238  |                |
| 4 | 0.00000011  |                |
| 5 | 0.00000000  |                |

**Discussion of Results**

The study established that both double-step and triple-step iteration schemes converge strongly to the fixed point of the operator under suitable conditions in Banach spaces. The convergence analysis showed that the iterative sequences are converging to a unique fixed point.

The results further revealed that the double-step iteration (S-iteration) scheme converges faster than many classical iterative methods due to its additional intermediate step. However, the triple-step iteration (Noor iteration) scheme exhibited an even faster rate of convergence, requiring fewer iterations to achieve the same level of accuracy.

The numerical findings supported the theoretical results, confirming that the triple-step method (Noor iteration) is more efficient and computationally effective than the double-step (S-iteration) scheme.

**Conclusion**

In this work, we have established convergence analysis and its rate of various iterative approaches, incorporating error terms, via the Zamfirescu operator in Banach spaces. The study concluded that both methods are reliable for fixed point approximation. Therefore, the analysis revealed that Noor iteration, which involves three steps, converges more rapidly than the S-iteration, which consists of two steps, with respect to the convergence rate.

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